Solution - Homework 6

1. (a) $f'(x) = 2x + 1$

   (b) $g'(t) = -\dfrac{1}{(t+3)^2}$

   (c) $h'(x) = \dfrac{1}{2\sqrt{x+4}}$

2. (a) $f'(x) = 33x^{32} + 48x^{11} + 12x^3 + \sqrt{42}$

   (b) $g'(y) = \cos y - \sin y + \sec^2(y)$

   (c) $h'(t) = (t^4 + 60)\cos t + 3t^2 \sin t$

   (d) $f'(s) = -\csc^2 x$

   (e) $h'(z) = \dfrac{(z^3 + z^2 + 3z + 1)\cos z - (3z^2 + 2z + 3)\sin z}{(z^3 + z^2 + 3z + 1)^2}$

   (f) $r'(x) = \frac{7}{2}x^{\frac{5}{2}} + \frac{9}{2}x^{\frac{1}{2}} + \frac{3}{2}x^{-\frac{1}{2}} = \dfrac{7x^3 + 9x + 3}{2\sqrt{x}}$

   (g) $F'(v) = 2v + 3 - 4v^{-2} = \dfrac{2v^3 + 3v^2 - 4}{v^2}$

   (h) $Q'(y) = -\dfrac{1}{(y-1)^2}$

   (i) $R'(m) = \frac{1}{5}\left(\dfrac{m^2}{secm}\right)^{-\frac{4}{5}} \cdot \dfrac{2m - m^2 \tan m}{\sec m} = \dfrac{m(2\cos m - m\sin m)}{5(m^2 \cos m)^{4/5}}$

   (j) $S'(p) = p \cdot (3p^2 + 4)^2 \cdot (p^3 + 2p^2 + 4)^4 \cdot (63p^3 + 96p^2 + 60p + 152)$

   (k) $T'(x) = \dfrac{2(2x-5)}{(x-2)^2(x-3)^2}$

   (l) $G'(z) = 3\cos^2(3z) - 3\sin^2(3z)$

   (m) $H'(t) = t^3(4\sin t \cos t + t(\cos^2 t - \sin^2 t)) = t^3\left[2\sin(2t) + t\cos(2t)\right]$

   (n) $f'(x) = \dfrac{x\cos(\sqrt{x^2+5})}{2\sqrt{x^2+5}\sqrt{\sin(\sqrt{x^2+5})}}$

   (o) $g'(t) = \dfrac{t^6 + 3t^2}{(t^4+1)^{\frac{3}{2}}}$

   (p) $p'(v) = (v^3+v+4)^4(2v^3+3v^2+4)^3\left[5(3v^2+1)(2v^3+3v^2+4) + 4(4v^2+6v)(v^3+4v+4)\right]$

   (q) $\phi'(x) = -\dfrac{3x^{\frac{2}{3}} + 1}{9x^{\frac{2}{3}}(x + x^{1/3})^{\frac{4}{3}}}$

(r) $\psi'(t) = 2\sin\left(\dfrac{t^3+1}{t^2+2t}\right)\cdot\cos\left(\dfrac{t^3+1}{t^2+2t}\right)\cdot\dfrac{t^4+4t^3-2t-2}{(t^2+2t)^2}$

(s) $f'(x) = \dfrac{\sec^2(\ln x)}{x}$

(t) * $y' = (1+\ln x)x^x$

(u) * $h'(x) = -\sin(x^{\sin x})\cdot x^{\sin x}\cdot\left[\dfrac{\sin x}{x} + \ln x\cos x\right]$

(v) $F'(t) = e^{t\cos^2 t}\cdot(\cos^2 t - 2t\cos t\sin t)$

(w) $y' = \dfrac{1}{2\sqrt{1+e^x\ln(x^2+1)}}\left[e^x\ln(x^2+1) + \dfrac{e^x\cdot 2x}{x^2+1}\right]$

(x) $p'(x)$

(y) $q'(x) = x\log_8 + \dfrac{x}{2ln8}$

(z) * $y' = \left[\sec x\tan x\ln x + \dfrac{\sec x}{x}\right]x^{\sec x}$

* means you will need to use logarithmic differentiation

3. (a) $\dfrac{dy}{dx} = \dfrac{3}{2}x^{\frac{1}{2}} + x^{-\frac{3}{2}}$; $\dfrac{d^2y}{dx^2} = \dfrac{3}{4}x^{-\frac{1}{2}} - \dfrac{3}{2}x^{-\frac{5}{2}}$

   (b) $\dfrac{dv}{du} = \dfrac{3}{5}u^{-\frac{2}{5}} - \dfrac{44}{7}u^{\frac{4}{7}}$; $\dfrac{d^2v}{du^2} = -\dfrac{6}{25}u^{-\frac{7}{5}} - \dfrac{176}{49}u^{-\frac{3}{7}}$

   (c) $\dfrac{dz}{dx} = -2x\cdot\sin(x^2)$; $\dfrac{d^2z}{dx^2} = -2\sin(x^2) - 4x^2\cdot\cos(x^2)$

4. (a) $y = 3x+2$

   (b) $y = 2x-1$

   (c) $y - 3 = -3\sqrt{3}\left(x - \dfrac{\pi}{3}\right)$

   (d) $y + x = \pi$

5. $$y' = \left[\frac{3}{x+1} + \frac{4}{x-3} - \frac{1}{4}\cot x\right]\frac{(x+1)^3(x-3)^4}{\sqrt[4]{\sin x}}.$$

6. $-\sin x$

7. $n!$

8. $x = -1, 5$

9.

10. (a) $\dfrac{dy}{dx} = x^n \cdot f^{'}(x) + nx^{n-1} \cdot f(x)$

    (b) $\dfrac{dy}{dx} = \dfrac{3x^2 \cdot f^{'}(x) + x \cdot f(x) - 2}{3x^{\frac{5}{3}}}$

    (c) $\dfrac{dy}{dx} = 5x^7 \cdot f^{'}(x^5) + 3x^2 \cdot f(x^5)$

    (d) $\dfrac{dy}{dx} = \dfrac{f^{'}(x)[g(x)]^2 + g^{'}(x)[f(x)]^2}{[f(x) + g(x)]^2}$

11. (a) $h^{'}(2) = 2$

    (b) $F^{'}(2) = 44$

12. $\dfrac{1}{2}$

13. (a) 4000

    (b) $\dfrac{1}{32}$